

# NONCOMMUTATIVE GEOMETRY, NEGATIVE PROBABILITIES AND CANTORIAN-FRACTAL SPACETIME

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## ABSTRACT

A straightforward explanation of the Young's two-slit experiment of a quantum particle is obtained within the framework of the Noncommutative Geometric associated with El Naschie's Cantorian-Fractal transfinite Spacetime continuum.

## 1. Introduction

One of the most paradoxical conclusions of QM is related to an indivisible quantum particle traversing the Young's two-slit experiment; i.e the coexistence of a point particle at two separate spatial locations and the origins of the wave-particle duality in QM [1]. It was argued in [1] that provided spacetime is effectively a four-dimensional random manifold and the path of a quantum particle is fractal then any question about the exact spatial location of a microscopic point is fundamentally undecidable due to the inherent uncertainty and fuzziness of the geometrical structure of such space [1]. This uncertainty is caused by an intrinsic indistinguishability between *intersections* and *unions* within the subsets of the random effectively four-dim manifold,  $\mathcal{E}_c^{(4)}$ . Such space is just one representative element of the infinite-dimensional Cantorian-Fractal spacetime transfinite continuum,  $\mathcal{E}_c^{(\infty)}$ . The latter continuum is just a representative of von Neumann's Noncommutative Geometry. Therefore a "point" within  $\mathcal{E}_c^{(\infty)}$  can in a sense occupy *two* different locations at the same time.

El Naschie considered from the start the backbone random Cantor set  $S_c^{(0)}$ , whose dimensionality with probability **one** equals the Golden Mean,  $\phi = (\sqrt{5} - 1)/2$ , as was shown by the Mauldin-Williams theorem [2] and extended to all of  $\mathcal{E}_c^{(\infty)}$  by [3]. In essence, these randomly constructed spaces are related to the random homeomorphisms which furnish a natural probability measure on the spaces of *all* the probability distributions. For a crucial role that the "probability on the space of all probability distributions" has in the construction of Bell's inequalities and in von Mises frequency approach to probability theory, versus the standard Kolmogorov approach, see [4].

The dimension of the *dual* or complementary set to  $S_c^{(0)}$ , relative to the normal set given by the space  $\mathcal{E}_c^{(1)}$ , of dimension 1, is:  $\tilde{d}_c^{(0)} = 1 - d_c^{(0)} = 1 - \phi = \phi^2$ . Using the well-known relation between the Hausdorff dimension of a fractal path and the Hurst exponent  $H$  one finds [1]:

$$d_{path} = \frac{1}{H} = d_c^{(2)} = \frac{1}{\phi} = 1 + \phi. \quad \tilde{d}_{path} = \tilde{d}_c^{(2)} = \frac{1}{1 - \phi} = \frac{1}{\phi^2} = (1 + \phi)^2. \quad (1)$$

El Naschie finally showed that if, and only if, there is an equivalence between *unions* and *intersections* in the concerned space then we must have:

$$d_{critical} = d_c^{(2)} + \tilde{d}_c^{(2)} = \frac{1}{\phi} + \frac{1}{\phi^2} = \frac{\phi(1 + \phi)}{\phi^3} = \frac{1}{\phi^3} = \frac{1}{\phi} \frac{1}{\phi^2} = d_c^{(2)} \tilde{d}_c^{(2)} = 4 + \phi^3. \quad 1 + \phi = \frac{1}{\phi}. \quad (2)$$

Where  $4 + \phi^3 = d_{critical} = \dim \mathcal{E}_c^{(4)} = 4.236067\dots$  The critical dimension coincides exactly with the Hausdorff dimension of the random set  $\mathcal{E}_c^{(4)}$  which is embedded densely onto a smooth set of topological dimension equal to four. The backbone random set of dimension  $d_c^{(0)} = \phi$ , a randomly constructed Cantor set, is embedded densely onto a set of topological dimension zero: a "point". This justifies the notation

:  $d_c^{(n)}$  is the Hausdorff dimension of a randomly constructed space embedded onto a smooth manifold of integer topological dimension equal to  $n$ .

In the next section we shall provide a probabilistic argument to El Naschie's results by firstly following in detail the particular Peano-Hilbert path traversing the two-slits in a fashion to be described below. This provides a very natural explanation of *negative* probabilities within the framework of Noncommutative Geometry. And furthermore why there is an asymmetry of the randomly constructed Cantor Set  $\mathcal{E}_c^{(4)}$  used by El Naschie in the Young's two-slit experiment.

Negative dimensions and Negative Entropies [5] were of crucial importance in the explicit numerical proof why the average spacetime dimension, over an infinity of dimensions ranging from  $-2$  to  $\infty$ , is of the order of  $4 + \phi^3$  [6]. This lends further credence that there is must be a deeper reason why we live in four dimensions than the one provided by the compactification schemes of string,  $M$  theory : these attempts are no explanation as to why we live in four dimensions. For a discussion of the history of negative probabilities, and for that matter complex probability in QM, see the report article by Muckenheimer [7]. For their role in  $p$ -Adic Quantum Mechanics see [8]. A detailed discussion of why there is **no EPR** paradoxes within the framework of the New Relativity Theory [9], linked to Cantorian-Fractal spacetimes, see [10].

## 2. Negative Probabilities and Noncommutative Geometry

The probabilistic extension of El Naschie's dimensional arguments to explain the Young's two-slit experiment within the framework of Cantorian-Fractal spacetime requires choosing the Peano-Hilbert path from the source to the detector traversing the two slits in a particular fashion. The source of particles (say and electron gun) is located far to the left of the partition with the two slits  $B, A$ . The slit  $B$  lies above the slit  $A$ . The detector lies far to the right of the partition. Region **I** is the one to the left of the partition. Region **II** is to the right.

Imagine the electron's path (from the source to the partition) impinging on the upper portion of slit  $B$  at an angle  $\alpha$  (which for simplicity we may take to be 45 degrees). Once it crosses slit  $B$  it experiences a zig-zag typical of a Peano-Hilbert geodesic fractal path in a *clockwise* fashion : the zig-zagging in region **II**, begins at point 1 with a sharp  $90^\circ$  turn to the left reaching the vertex 2, where another  $90^\circ$  turn to the right takes the electron to the vertex 3. Another two consecutive  $90^\circ$  turns will take the particle from 3 back to the beginning 5, where the curve 12345 almost "closes" at points 1, 5 inside the slit  $B$ .

To sum up : The traversal of the first iteration of the Peano-Hilbert geodesic curve in region **II** to the right of the slit  $B$  is performed *clockwise*.

Once back in slit  $B$  the path zig-zags towards the point 6 in region **I** in a south-west bound fashion, at an angle perpendicular to the initial impinging direction to the partition (south-east). At vertex 6, in region **I**, it zig-zags in a *counterclockwise* fashion beginning the second iteration of a Peano-Hilbert fractal geodesic path moving towards the lower edge of slit  $A$ . It crosses the lower edge of slit  $A$  and reaches vertex 7 when it performs a  $90^\circ$  turn to the left reaching the vertex 8. Another counterclockwise turn from 8 back to the lower edge of slit  $B$ . It crosses slit  $B$  **again** at vertex 9 towards region **I**. A further  $90^\circ$  turn at vertex 9, south-west bound towards vertex 6, and another  $90^\circ$  turn, to cross finally the upper edge of the slit  $A$ , reaching the detector in region **II** at an angle  $\alpha = 45^\circ$ .

To sum up : The traversal of the second iteration of the Peano-Hilbert geodesic curve in regions **I** and **II** surrounding slits  $A$  and  $B$  is performed *counterclockwise*.

Notice the crucial difference between the two Peano-Hilbert geodesic paths. In the first case it exists to the right of the slit  $B$  while in the second case it exists in both regions **I**, **II** of the partition and it winds around both slits  $A, B$ . This essential difference will account for the numerical results that follow.

We will assign probability magnitudes to the regions at  $A, B$  respectively :

$$|p_A| = |p(A)| = \frac{1}{d_A} = \phi. \quad |p_B| = |p(B)| = \frac{1}{d_B} = \phi^2. \quad (3)$$

The physical explanation of this goes as follows : The region around slit  $B$  is comprised of 3 lines. One line is right-flowing and two lines are left flowing. The region around slit  $A$  is comprised of only two right-flowing lines. It is natural to assign a *higher* dimensionality to the region around slit  $B$  than the one around  $A$ . The right flowing line in slit  $B$  could correspond to an *upper* bridge connecting regions **I**, **II**. And

the two left-moving lines flow along a *lower* bridge connecting regions **II**, **I** ( like a knot). In slit *A* there is only one bridge ( where the two right-moving lines flow from **I** to **II** ).

Furthermore, in slit *B* the net flux is *negative* while in slit *A* is *positive*. In slit *B* there are two negative units of flux ( left moving) and one positive unit of flux ( right moving) giving a net flux of  $1 - 2 = -1$  ( left moving). In region *A* there are two positive units of right moving flux so net flux is 2 ( positive).

For these reasons we will assign *different* magnitudes and signs to  $p_B$  w.r.t  $p_A$  :

$$p_A = p(A) = \frac{1}{d_A} = \phi. \quad p_B = p(B) = -\frac{1}{d_B} = -\phi^2. \quad (4)$$

The Noncommutative nature of Cantorian-Fractal spacetime [1] is expressed explicitly as :

$$p(A \wedge B) = -p(B \wedge A) = (\phi)(-\phi^2) = -\phi^3. \quad (5)$$

Using the standard definitions of conditional probabilities :

$$p(A|B) = \frac{p(A \wedge B)}{p(B)} = \frac{(-\phi^3)}{(-\phi^2)} = \phi. \quad (6a)$$

$$p(B|A) = \frac{p(B \wedge A)}{p(A)} = \frac{(\phi^3)}{(\phi)} = \phi^2. \quad (6b)$$

$p(A|B)$  is the conditional probability for *A* provided *B* **has** occurred.  $p(B|A)$  is the conditional probability for *B* provided *A* **has** occurred.  $p(A|B)$  is associated with the first iteration of the Peano-Hilbert geodesics . It is the probability that the Peano-Hilbert geodesic which left slit *B* will zig-zag around point 6 and enter the slit *A* below. And vice versa,  $p(B|A)$  is associated with the probability that the second iteration of the Peano-Hilbert geodesic, around slit *A*, will wind around slit *B* after having gone through slit *A*.

However,  $p(A \wedge B) = -p(B \wedge A) = -\phi^3$  is the joint probability associated with both events *A*, *B* to take place and coexist **simultaneously**. This is possible in a cantorian-Fractal spacetime which is in essence a “ Noncommutative Pointless” geometry. The Noncommutative nature of the Cantorian-Fractal spacetime  $\mathcal{E}_c^{(\infty)}$  is the one responsible for the fact that  $p(A \wedge B) = -p(B \wedge A)$ .

To check that negative probabilities do **not** violate *unitarity* we must verify that the sums of all probabilities does not exceed unity. In fact the net sum equals **unity** exactly ! :

$$p(A) + p(B) + p(A \wedge B) + p(A|B) + p(B|A) = [\phi - \phi^2 - \phi^3] + [\phi + \phi^2] = 1. \quad (7a)$$

Since

$$[\phi - \phi^2 - \phi^3] = \phi - \phi^2(1 + \phi) = \phi - \frac{\phi^2}{\phi} = 0. \quad [\phi + \phi^2] = \phi(1 + \phi) = 1. \quad (7b)$$

Is this all nothing but a numerical coincidence or design that we live in  $4 + \phi^3$  [6] ? And that  $4 + \phi^3$  is also the average dimension of the Cantorian-Fractal spacetime  $\mathcal{E}_c^{(\infty)}$  :  $< \dim \mathcal{E}_c^{(i)} > = \dim \mathcal{E}_c^{(4)} = 4 + \phi^3 = 4.236067... [1]$  ? .Where *i* ranges from  $-\infty$  to  $\infty$ .

Concluding, the Noncommutative nature of Cantorian-Fractal spacetime [1] combined with the notion of the inherent *negative* probability assigned to a Peano-Hilbert “ loop “ around the region of slit *B*, because its orientation is *opposite* to the Peano-Hilbert “ loop” located in the region around slit *A*, explains in a very straightforward ( but **non-classical**) fashion the wave-particle duality of an indivisible quantum-particle. The Noncommutative Cantorian-Fractal spacetime is **not** a classical space. This supports further the interpretation of the Schrodinger equation by Nagasawa as *two dual* diffusion equations [5] , one forward and the other backwards in time , which also agrees with Nottale’s [5] fractal geodesics behaviour of the Young’s two-slit experiment and El Naschie complex time and G. Ord’s spiral gravity models [1].

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